

MODEL-FREE INTERVAL-BASED LOCALIZATION IN MANETS

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ABSTRACT

Self-localization of sensor nodes has become a fundamental requirement for many sensor networks applications. In this paper, we propose an interval-based rings-overlapping technique using the comparison of the received signal strength indicators. The high performance of this method remains in the way that it avoids the estimation of the channel pathloss model. Compared to the guaranteed boxed localization based on connectivity measurements, the proposed method is robust under irregular radio propagation patterns. Simulation results corroborate the efficiency of this method in terms of accuracy and computation time.

Index Terms— Mobile sensor networks, interval analysis, mobility model, RSSI comparison, ring overlapping

1. INTRODUCTION

Mobile Ad hoc sensor NETWORKS (MANETs) have currently become a popular and challenging research field. They are composed of a distributed collection of smart sensors, each of which has sensing, computation and communication capabilities. The major limitations of sensor networks are their limited memory resources and energy reserve. The sensors batteries are in fact not renewable and have consequently limited lifetimes. Many applications involve mobile sensors such as environment monitoring and target tracking [1]. Almost all applications require information about the geographical locations of the sensors.

Location estimation is an important task in many innovative applications of mobile sensor networks. For this purpose, many localization algorithms have been proposed. Equipping each sensor with localization hardware such as GPS represents a high energy consuming and expensive solution [2]. An alternative solution consists of providing some sensors (denoted *anchors*) with GPS and localizing other sensor nodes using exchanged information with anchors. Many anchor-based approaches have been proposed in literature. Some existing works propose to perform successive static localization

algorithms [3, 4]. These techniques compute the position of nodes, based only on the information communicated with the anchors. Recently, Monte-Carlo techniques have been proposed [5, 6]. These Bayesian filtering methods are based on an approximate sequential Monte-Carlo method where particles are drawn to estimate node locations. They outperforms static localization since they take advantage of the temporal correlation of the mobile node trajectory to improve the accuracy of the localization.

In our previous work [7], we proposed a Guaranteed Boxed Localization (GBL) technique based on interval analysis. It deals with intervals as a new kind of numbers where all arithmetic and set operations can be performed [8, 9, 10]. The GBL method consists of using a dynamic mobility model that propagates the location incertitude in an interval form. The localization problem is then defined as a Constraint Satisfaction Problem (CSP) and the Waltz algorithm [11] is used in order to solve the CSP where both observations and propagation equations define the constraints. In [7], the localization is performed using connectivity measurements based on the pathloss model. It consists of a radio propagation model that predicts the distance traveled by the signal given the initial and the received signal strengths. In this paper, we propose an enhanced interval-based method using the comparison of the Received Signal Strength Indicators (RSSI). It performs the localization process using rings overlapping [12]. One essential advantage of the method is that it overcomes the high correlation of the performance of the GBL method to the communication range value. Using RSSI comparison, the method does not need the knowledge of the channel pathloss parameters. Beside its guaranteed low cost aspect, the proposed algorithm yields a robust range-free localization under irregular radio propagation patterns.

The paper is organized as follows. In Section 2, we briefly introduce the interval analysis theory. In Section 3, we describe the proposed model-free localization technique in MANETs. Section 4 reports simulation results while Section 5 concludes the paper.

2. INTERVAL ANALYSIS THEORY

Interval analysis is a broad field that treats an interval as a new kind of numbers [8, 9]. Since its introduction, the interval approach has undergone a rapid and wide development. It is based on the simple idea of enclosing a set of real numbers with an interval. The purpose of interval analysis is to provide guaranteed solutions to non-linear optimization problems. Its guaranteed aspect consists of bounding errors and uncertainties on computed quantities.

2.1. Introduction to intervals

An interval, denoted $[x]$, is defined by a closed bounded set of real numbers as follows,

$$[x] = [\underline{x}, \bar{x}] = \{x \in \mathbb{R} \mid \underline{x} \leq x \leq \bar{x}\} \quad (1)$$

where \underline{x} and \bar{x} are lower and upper bounds of the interval $[x]$. An interval could also be regarded as a number represented by the ordered pair of its endpoints \underline{x} and \bar{x} . The width of an interval is defined as $w([x]) = \bar{x} - \underline{x}$. A box, denoted $[x]$, is a multi-dimensional interval enclosing vectors. It is defined by the Cartesian product of n real intervals as follows,

$$[x] = [x_1] \times \cdots \times [x_n] = [\underline{x}_1, \bar{x}_1] \times \cdots \times [\underline{x}_n, \bar{x}_n]. \quad (2)$$

The width of the box is the largest width of all its intervals. Consider a box $[x]$ of \mathbb{R}^2 . The area of $[x]$ is the product of the widths of its two intervals as shown in the following,

$$\mathbb{A}([x]) = w([x_1]) * w([x_2]) = (\bar{x}_1 - \underline{x}_1) * (\bar{x}_2 - \underline{x}_2). \quad (3)$$

An interval has a dual nature as both a number and a set of real numbers. Consequently, all arithmetic and set operations are developed for intervals, for instance the inclusion (\subset), the intersection (\cap), the interval union (\sqcup), the addition ($+$), the subtraction ($-$), the multiplication ($*$), the division (\div) and others. If $[x] = [\underline{x}, \bar{x}]$ and $[y] = [\underline{y}, \bar{y}]$ then the intersection of $[x]$ and $[y]$ is empty, $[x] \cap [y] = \emptyset$, if $\bar{x} < \underline{y}$ or $\underline{x} > \bar{y}$. Otherwise, the intersection is an interval defined as follows,

$$[x] \cap [y] = [\max(\underline{x}, \underline{y}), \min(\bar{x}, \bar{y})]. \quad (4)$$

The union of two intervals is not, in general, an interval. We define the interval union \sqcup as the smallest interval containing the set union as follows,

$$[x] \sqcup [y] = [[x] \cup [y]] = [\min(\underline{x}, \underline{y}), \max(\bar{x}, \bar{y})]. \quad (5)$$

Let $[x] = [1, 3]$ and $[y] = [5, 7]$. The interval union of $[x]$ and $[y]$ is thus given by $[x] \sqcup [y] = [[1, 3] \cup [5, 7]] = [1, 7]$.

In the same manner as for the set operations, the elementary arithmetic operations can be characterized as follows,

$$[x] * [y] = \{x * y \mid x \in [x] \text{ and } y \in [y]\} \quad (6)$$

where $*$ \in $\{+, -, *, \div\}$. The interval arithmetics are thus defined individually in the following equations,

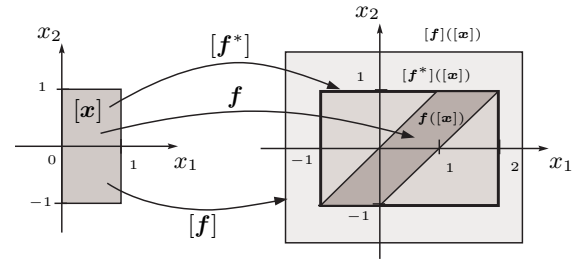


Fig. 1. Inclusion functions and wrapping effect.

$$[x] + [y] = [\underline{x} + \underline{y}, \bar{x} + \bar{y}], \quad (7)$$

$$[x] - [y] = [\underline{x} - \bar{y}, \bar{x} - \underline{y}], \quad (8)$$

$$[x] * [y] = [\min(\underline{x} * \underline{y}, \underline{x} * \bar{y}, \bar{x} * \underline{y}, \bar{x} * \bar{y}), \max(\underline{x} * \underline{y}, \underline{x} * \bar{y}, \bar{x} * \underline{y}, \bar{x} * \bar{y})], \quad (9)$$

$$\frac{1}{[x]} = \left[\frac{1}{\bar{x}}, \frac{1}{\underline{x}}\right] \text{ if } \underline{x} > 0 \text{ or } \bar{x} < 0, \quad (10)$$

$$[x] \div [y] = [x] * \frac{1}{[y]}. \quad (11)$$

Another interesting tool in interval analysis is the inclusion function. Consider a function f defined from \mathbb{R}^n to \mathbb{R}^m . The interval function $[f]$ from \mathbb{R}^n to \mathbb{R}^m is an inclusion function of f if

$$\forall [x] \in \mathbb{R}^n, f([x]) \subseteq [f]([x]). \quad (12)$$

The purpose of the inclusion function is to provide enclosers for $f([x])$. It is obvious that a function can have an infinite number of inclusion functions. The minimal inclusion function of f , denoted $[f^*]$, is the smallest box that encloses it. Let f be defined from \mathbb{R}^2 to \mathbb{R}^2 by $f(x) = \begin{pmatrix} x_1 + x_2 \\ x_2 \end{pmatrix}$ where $x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $x_1 \in [0, 1]$ and $x_2 \in [-1, 1]$. Figure 1 shows two inclusion functions, one of them being the minimal inclusion function of f . We can see that the area of the enclosing box is higher than the exact solution. This is the so-called *wrapping effect*.

2.2. Waltz contractor

Interval analysis provides powerful tools to resolve Constraint Satisfaction Problems (CSPs), such as non-linear optimization problems. A CSP consists of a set of constraints $f_i, i = \{1, \dots, m\}$ defined from \mathbb{R}^n to \mathbb{R} . Satisfying the constraints consists of finding the set of solutions S whereas $S = \{x \in \mathbb{R}^n \mid f_i(x) = 0, 1 \leq i \leq m\}$. Resolving the CSP in the interval framework consists of finding the minimal box $[x^*]$ that contains the solution set S . The power of interval arithmetics lies in their capacity to provide rigorous boxes that enclose the ranges of operations and functions. Applied in CSP resolutions, they yield interval results that contain the entire set of possible values of the solution.

The main shortcoming of the interval analysis tools remains in their incapacity to enclose the solution set with the minimal box. Consequently, a major focus of interval analysis turned to develop practical interval algorithms that contract the solutions. A contractor is, thus, an algorithm that narrows as possible the bounds of the solution box. Many contractors have been proposed in literature [9, 10]. One forward-backward technique is the Waltz contractor [11]. It consists of a simple algorithm that propagates iteratively the constraints over a prior box, without any prior order, until no more contraction is performed. It is worth noting that the Waltz contractor is an efficient low-cost algorithm. However, being an iterative method, it may yield local minimum.

3. BOXED LOCALIZATION BASED ON RSSI COMPARISON

A sensor field consists of two types of sensors: *nodes* and *anchors*. Nodes are sensors with unknown positions, whereas anchors are equipped with GPS or other positioning systems, and thus know their exact locations. The nodes and the anchors are able to move uncontrollably in the network. The problem is to estimate the nodes locations given the positions of the anchors and some measurements information. In the following, we will consider the localization of one mobile node, without loss of generality, since each node localizes independently to others. The same algorithm is implemented on the remaining mobile nodes.

3.1. Mobility model of the node

In mobile node localization, we are interested in estimating the location of the node at ever time-step. Besides the measurements to anchors, the estimation problem takes advantage of the motion of the node to improve the accuracy. Our goal is to provide a mobility model that is general enough to accommodate a large number of real applications. The latter point is motivated by many definitions in literature about simple mobility models [13]. In our algorithm, we assume that a node is unaware of its moving speed and direction. The only available prior information is that its speed is less than a maximal velocity, denoted v_{max} . The mobility equation is as follows,

$$(x_1(t) - x_1(t-1))^2 + (x_2(t) - x_2(t-1))^2 \leq v_{max}^2 \quad (13)$$

where x_1 and x_2 are the coordinates of the mobile node. So, if $\mathbf{x}(t-1)$ is the punctual position of the node at time $t-1$, the current solution $\mathbf{x}(t)$ is contained in the disk centered on the previous position having v_{max} as radius. This mobility model is in fact a generic model that does not suppose any prior knowledge of the direction and the speed variation of the node. More details about the mobility of the node could be added to refine the motion model, yielding more accuracy in the localization process.

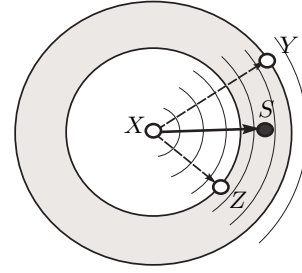


Fig. 2. Ring generation using RSSI comparison.

3.2. Rings generation using RSSI comparison

The general idea of our technique is to overlap rings centered at anchors. The rings are generated by comparison between the RSSI signals an anchor sends to the mobile node and the remaining anchors in its vicinity. This technique assumes that with the increase of the distance between a sender and a receiver, the signal strength monotonically decreases. The inner (resp. outer) radius of a ring centered at an anchor is the maximal (resp. minimal) distance from this anchor to the set of anchors which RSSI signals are higher (resp. lower) than the RSSI received by the mobile node to be localized. Suppose we have three anchors X , Y and Z and one mobile node S . If $RSSI_{XY} \leq RSSI_{XS} \leq RSSI_{XZ}$ then $d_{XZ} \leq d_{XS} \leq d_{XY}$. Thus, the node S lies inside the ring defined by d_{XZ} and d_{XY} as inner and outer radii respectively (see Figure 2). More generally, the ring number i , $i \in \{1, \dots, M\}$, is centered at the anchor i and has r_i and R_i as inner and outer radii defined as follows,

$$\begin{cases} r_i = \max_{j \neq i} \{ d_{ij} \mid RSSI_{ij} \geq RSSI_i \} \\ R_i = \min_{j \neq i} \{ d_{ij} \mid RSSI_{ij} \leq RSSI_i \} \end{cases} \quad (14)$$

where M is the number of anchors, d_{ij} is the Euclidean distance between the anchor i and the anchor j , $RSSI_{ij}$ is the signal strength sent by the anchor i and received by j and $RSSI_i$ is the strength of the signal sent by the anchor i and received by the unknown-position mobile node. An illustration is shown in Figure 3. The observation equations are thus set by the following,

$$r_i^2 \leq (x_1(t) - a_1^i)^2 + (x_2(t) - a_2^i)^2 \leq R_i^2, \quad i \in I \quad (15)$$

where a_1^i and a_2^i are the coordinates of the i -th anchor and I is the indices set of all anchors involved in the localization process. I could correspond to $\{1, \dots, M\}$, which is the set of all anchors in the network, as well as to the set of anchors within the communication range of the node.

3.3. RSSI comparison based localization

The localization problem is defined as a Constraint Satisfaction Problem (CSP) where the mobility model is used to propagate the previous position to the current time. The main idea of this technique is to define estimated locations as two-dimensional boxes, denoted $[\mathbf{x}]$. The prior model assuming

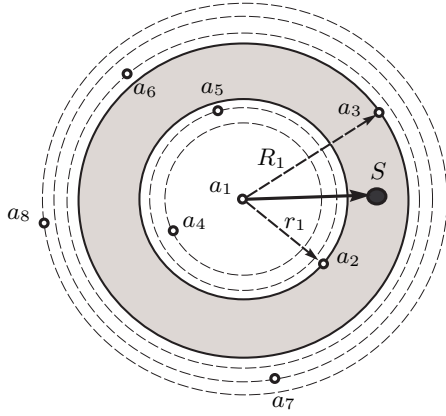


Fig. 3. Definition of the inner and outer radii of a ring.

that sensors move with a maximal velocity v_{max} is consistently incorporated with the measurement equations. In an interval framework, the problem consists of propagating the previous box using the motion equation and then of contracting the resulting box with the Waltz algorithm using the observation rings as constraints. The motion equation is defined as follows,

$$([x_1](t) - [x_1](t-1))^2 + ([x_2](t) - [x_2](t-1))^2 = [0, v_{max}^2] \quad (16)$$

while measurements are given by

$$([x_1](t) - a_1^i)^2 + ([x_2](t) - a_2^i)^2 = [r_i^2, R_i^2], \quad i \in I \quad (17)$$

where $[x_1](t)$ and $[x_2](t)$ are the coordinate intervals of the mobile node at time t , a_1^i and a_2^i are the coordinates of the i -th anchor, r_i and R_i are the radii of the ring centered on the anchor i and I is the set of indices of the anchors used to localize the mobile node at time t .

The localization problem, put in an interval form, is then solved as a CSP defined by the set of constraints above. Solving a CSP using intervals consists of finding the minimal box that satisfies all the constraints. Propagating boxes allows a guaranteed estimation of the node position using only few parameters. The Waltz algorithm provides a solution of the CSP. It is a simple technique based on propagating iteratively all constraints without any prior order until no contraction is possible. This method allows to find local minimal boxes with an effective time cost.

Each localization step is divided into a propagation phase and a correction phase. In the propagation phase, the node uses its mobility model to propagate its previous position, yielding a prior position box. In the correction phase, both measurements and mobility equation are incorporated in the Waltz algorithm in order to contract the prior box. The interval equations (16) and (17) are reformulated as primitive equations where each variable is defined as a function of all others, in order to be implemented in the contractor. Equation

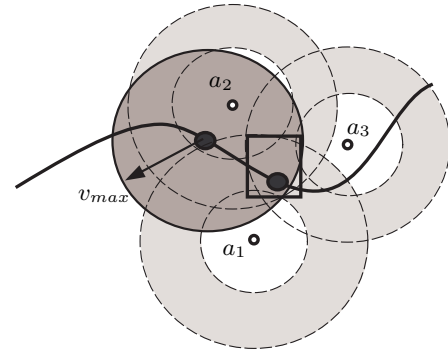


Fig. 4. Interval-based localization using RSSI comparison.

(16) yields the following,

$$[x_1](t) = [x_1](t-1) + [-[b_1](t) \sqcup [b_1](t)], \quad (18)$$

$$[b_1](t) = [\sqrt{v_{max}^2 - ([x_2](t) - [x_2](t-1))^2}], \quad (19)$$

$$[x_2](t) = [x_2](t-1) + [-[b_2](t) \sqcup [b_2](t)], \quad (20)$$

$$[b_2](t) = [\sqrt{v_{max}^2 - ([x_1](t) - [x_1](t-1))^2}], \quad (21)$$

while equation (17) gives the following primitives,

$$[x_1](t) = [x_1](t) \cap [a_1^i + [-[b_1^i](t) \sqcup [b_1^i](t)]], \quad (22)$$

$$[b_1^i](t) = [\sqrt{R_i^2 - [a_2^i - [x_2](t)]^2}], \quad (23)$$

$$[x_2](t) = [x_2](t) \cap [a_2^i + [-[b_2^i](t) \sqcup [b_2^i](t)]], \quad (24)$$

$$[b_2^i](t) = [\sqrt{R_i^2 - [a_1^i - [x_1](t)]^2}]. \quad (25)$$

An illustration of the method is shown in Figure 4. It shows the smallest box that could be obtained with our localization technique. It is worth noting that we are considering, for simplicity, a punctual previous position.

In order to reduce the computational cost of the localization process, the mobility and observation models could be approximated. The disk equations are thus relaxed to square equations. The first approximated scheme consists of a partially approximated one where the mobility equation is relaxed as follows,

$$\begin{cases} [x_1](t) = [x_1](t-1) + [-v_{max}, v_{max}] \\ [x_2](t) = [x_2](t-1) + [-v_{max}, v_{max}] \end{cases} \quad (26)$$

The second scheme consists of a fully approximated one where both the mobility equation and the observations are relaxed. The observation equations are thus written as follows,

$$\begin{cases} [x_1](t) = [x_1](t) \cap [a_1^i + [-R_i, R_i]] \\ [x_2](t) = [x_2](t) \cap [a_2^i + [-R_i, R_i]] \end{cases}, \quad i \in I. \quad (27)$$

The relaxed schemes yield larger boxes than the one obtained with the correct equations.

4. SIMULATIONS

In order to evaluate the effectiveness of our algorithm on MANETs, we moved a mobile node in a $100m \times 100m$

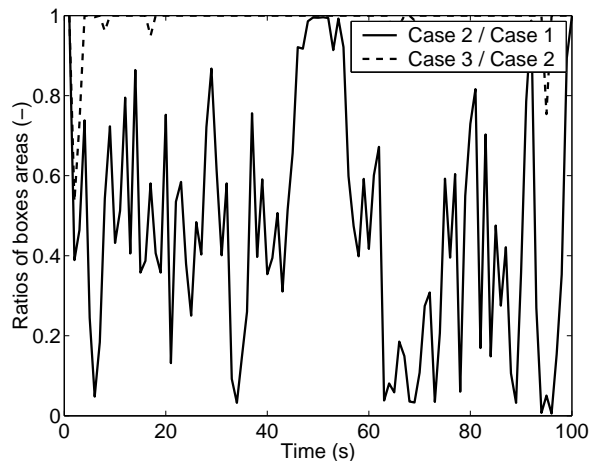


Fig. 5. Ratios of areas of estimated boxes: Case 2 over Case 1 in solid line and Case 3 over Case 2 in dashed line.

square area over 100s. The node trajectory is composed of two sinusoids with an abrupt direction change as shown in Figure 6. The maximal velocity of the mobile node is equal to $2.0343m.s^{-1}$. We also deployed anchors in the network in such a way that at least three anchors are within the communication range of the mobile node at every time step. In Section 3, we have considered two approximated schemes in addition to the original proposed method. The three proposed techniques are denoted as follows: Case 1 (fully approximated scheme), Case 2 (partially approximated scheme) and Case 3 (correct models). To evaluate the different methods, we define the error as the difference between the real positions and the center of the computed location boxes. The computation times needed to accomplish the localization for the three cases are respectively $0.4520s$, $0.8730s$ and $1.4510s$ whereas the average errors are respectively $1.7665m$, $0.9346m$ and $0.9316m$. Note that only anchors within the communication range of the node are taken into consideration in the localization process. Figure 5 shows the ratio of the areas of the boxes obtained in Case 2 over the areas obtained in Case 1 in solid line, and the ratio of the areas obtained in Case 3 over the areas obtained in Case 2 in dashed line. It is obvious that the relaxation of the observation model yields much larger boxes; while the approximation of the mobility model affects slightly the localization process in our example. Consequently, we will use Case 2 of the method in the following simulations. In the following, to validate the performances of our method, we compared it to the GBL technique that uses connectivity measurements to anchors [7]. We also compared our method to a Monte-Carlo based technique that uses the RSSI-comparison as observation model.

4.1. Comparison to the GBL method

In this section, a comparison of our method to the Guaranteed Boxed Localization (GBL) proposed in [7] is performed. Without loss of generality, we consider the localization of

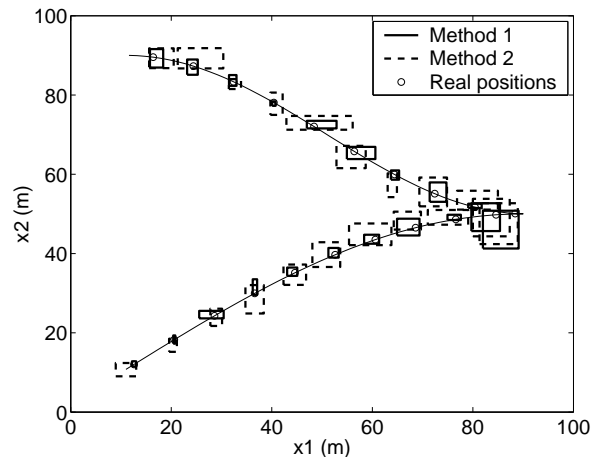


Fig. 6. Estimated boxes obtained with our method (Method 1) and the guaranteed boxed method (Method 2).

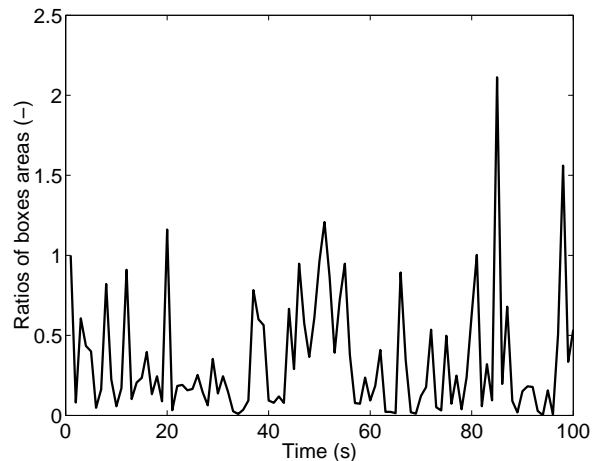


Fig. 7. Ratios of boxes areas obtained with our method over the areas obtained with the guaranteed boxed method.

one moving node since each node self-localizes independently from other nodes using only anchors information. In a $100m \times 100m$ area, we move the mobile node on a trajectory composed of two sinusoids over 100s with a maximal velocity equal to $2.035m.s^{-1}$. The sensing range is set to $10m$. Figure 6 illustrates the estimated boxes obtained with our method (Method 1) compared to the boxes computed with the GBL method (Method 2). As expected, the boxes in our method are smaller than the one resulting from the GBL technique. The average error obtained in our method is equal to $0.9346m$ while it is equal to $1.7775m$ with the GBL method. The computation times are equal to $0.9360s$ and $0.7020s$ in Method 1 and Method 2 respectively. In both methods, only the anchors within the communication range of the node are used to generate observation information. Despite the small increase of the computational time, the model-free RSSI-based method reduces the average boxes area to $11.9574m^2$ while it is equal to $35.8355m^2$ in the GBL method. Figure 7 shows the ratios of the areas of the boxes obtained in our method over those obtained in the GBL

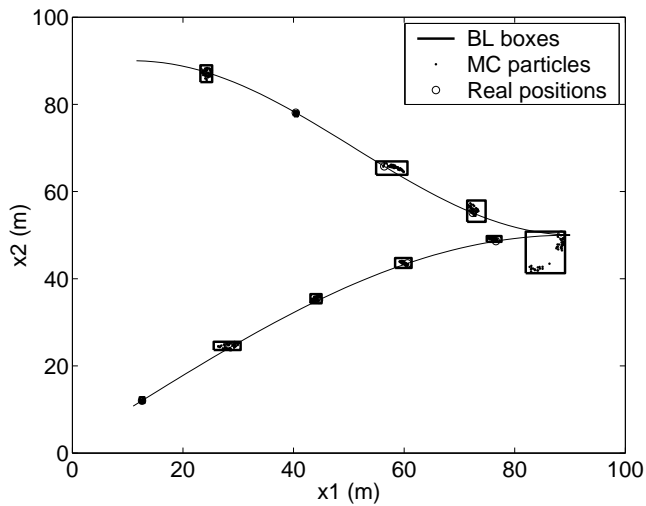


Fig. 8. Comparison of our method (BL) to a Monte-Carlo based method (MC).

method. The average ratio, equal to 0.3442, corroborates the efficiency of our method.

4.2. Comparison to a Monte-Carlo based method

The Monte-Carlo localization boxed method proposed in [6] consists of two phases: a prediction phase and an update phase. In the prediction phase, a square region is created using approximated mobility and connectivity models. Particles are then generated within this box. In the update phase, correct measurements are incorporated to filter and update data. Only particles satisfying the correct constraints are accepted. The process is iterated until a certain number of particles is kept. The observation model implemented consists of connectivity measurements. In this section, we are defining a Monte-Carlo based method that uses rings overlapping for observation. For this purpose, we modified the Monte-Carlo localization boxed method in order to be implemented by our observation model. The computation time needed by the Monte-Carlo method is equal to 46.6990s with 1.3426m as average error. Figure 8 shows the particles obtained with the Monte-Carlo method. The number of particles is set to 50. It shows the boxes obtained with our method, as well. Besides the important gain in computation time, the Monte-Carlo method requires the storage of at least 50 particles every time step while our method only needs to save the bounds of the coordinates describing one estimated box.

5. CONCLUSION

In this paper, we presented an interval-based localization method using RSSI-comparison of exchanged signals between anchors and between anchors and mobile nodes. The proposed method generates small intersection area and thus accurate location estimation. With the RSSI comparison, the

localization consists of a robust and non parameterized technique with a small computational time. Simulation results show that our method compares well to the GBL technique in terms of accuracy. It outperforms the Monte-Carlo based techniques in performance and computational costs, as well. In future works, we will deal with inaccurate environment and sensor failure problems. In addition, we will make use of the information exchanged between mobile nodes to improve the localization process.

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